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A COLLISION-AVOIDING MECHANISM BASED ON A THEORY OF MIND

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We develop a collision-avoiding mechanism for a system of individual agents (pedestrians) that move in a crowd trying to reach their different goal points. The agents avoid collisions on the basis of a model of the other agents' behavior, a "theory of mind," which is realized at different levels through an iterative process (the first level, or level 0, corresponds to ignoring the other agents' behavior, level 1 to assuming that the other agents will ignore each other, and so on).

The model is conceived in order to perform an evolutionary simulation of some basic parameters that determine the agent's sensorial, cognitive and behavioral system (the perception of the agent's own size, the attraction to the goal, the radius and angle of the field of view and the level of the theory of mind).

In this preliminary work we present our model, show that it reproduces some of the simplest organized behaviors of a system of pedestrians, and focus on some features of the theory of mind, as the difference between odd and even levels.

Keywords: Theory of mind; evolutionary simulation; genetic algorithm; crowd simulation.

1. Introduction

An "individual" (for example, an animal) provided with a sensorial system uses the information that it obtains about the surrounding environment to reach its goals (reproduction, alimentation, escape, etc.). When other individuals (prey, predators, and other predators chasing the same prey and also individuals belonging to the observer's same species) are perceived in its surroundings, we could expect this individual to take into consideration the possible actions of the perceived individuals, i.e. we could expect it to have at least a rough model of their behavior, which implies some level of knowledge about the aims of these other individuals and about the methods they use to realize those aims.

If this model of the others' behavior (a "theory of mind," or ToM) is complex enough, it should include the fact that the other individuals too are provided with a sensorial system and thus are able to obtain information about the environment and to use it. It is clear that a recursive situation could arise: since in the environment observed by the observed individual other individuals (including the observer itself) could be present, an appropriate model of their behavior could assume that the observed ones also have a ToM.

In this case we can say that the observer has a ToM of level 2 (level 0 corresponds to ignoring the others' behavior, i.e. not to having a ToM, and level 1 to assuming that the others do not have a ToM). Assuming that the others have a level 2 ToM implies having a level 3 ToM, and so on.

High ToM levels are surely present in human behavior, in particular when complex social interactions are concerned, and to reach his own aims a person needs to predict the other people's actions, which are the result of a network of reciprocal interactions and predictions (humans have a ToM of level 4 or level 5, according to Ref. 1).

Many researches have been devoted to understanding which animal species could have a ToM. According to recent works some animals, such as dogs [2], dolphins [3], goats [4] and crows [5], are able to follow the gaze of other individuals. Other researches seem to show the presence of a limited ToM in primates, but no indisputable signs have been found. Furthermore, this ability seems to be reduced just to the first two levels, and actual recursive thinking could be exclusive of human beings.

Takano, Katō and Arita [6, 7] have performed an evolutionary simulation in which a ToM-based prediction of the movement of the other agents was used to avoid collisions. That idea is at the basis of the crowd dynamics model that we present in this paper. In our model each agent is represented by a rigid disk that undergoes elastic collisions with other disks and walls. It has a goal and its fitness (which will be used in a future work to perform an evolutionary simulation on the parameters of the model) has a positive term given by the inverse of time needed to reach the goal plus a negative one determined by the momentum exchanged in bounces (a representation of "pain"). Both the sensorial system (radius and angle of sight) and the decision mechanism (perception of its own size, attraction to the goal and the ToM level that determines the prediction of the other agents' motion) are treated as evolvable parameters.

2. Description of the Model

Our aim is to perform an evolutionary simulation of pedestrians in a crowd, avoiding collisions while moving toward a given goal, with particular stress on the mechanism of prediction of the movement of the other pedestrians, which will be modeled through the concept of ToM.

For this purpose we introduce an idealized collision-avoiding mechanism in which a few free parameters can be optimized by a genetic algorithm, according to a fitness function in which a positive term is given by the velocity to reach the goal, and a negative one by collisions. Since the outcome of our model will be given by evolution corresponding to a very simple fitness function, we do not claim that it can describe the actual human behavior, but we expect it to present at least at a qualitative level some of the features of the self-organized motion that are present in actual crowd dynamics (see Refs. 8 and 9).

Furthermore, the use of a given decision mechanism with some evolvable parameters (instead of a completely evolved system, as a neural network) is suitable for a clearer interpretation of the results and for the introduction of concepts like ToM.

All the agents–pedestrians in our model are represented by hard disks in two dimensions that undergo elastic collision between them and with the walls (their physical dynamics is exactly integrated with an event-driven algorithm).

At each time step Δ_t the decisional mechanism is applied simultaneously by all the agents, on the basis of their goal and of their sensorial perception of the other agents. The output of the decisional mechanism is an impulsive force $\vec{f}$ that modifies the motion of the agent according to

$$\vec{v}(t) = \vec{v}(t - \Delta_t) + \vec{f}(t)\Delta_t, \tag{1}$$

$$\vec{x}(t + \Delta_t) = \vec{x}(t) + \vec{v}(t)\Delta_t \tag{2}$$

(actually, in our model the agents cannot pass a maximum velocity $v_{\max}$, which is imposed as a constraint on these equations).

The agents are split into two groups with different goals, each group moving in a corridor. The two corridors form a crossroads with an angle α (Fig. 1). (Actually, our model does not impose any constraint on the goals of each individual agent, but the latter choice leads to an easier interpretation of the results and to a comparison with the behavior of actual pedestrians.)

The goals are realized with a constant driving force term $\vec{f_g}$ directed along the corridors, while a tendency to avoid the collisions with the walls is introduced as a term $\vec{f_w}$ whose direction is normal to the walls and whose intensity is given by

$$f_w = \begin{cases} 0 & \text{if } x > d, \\ c\,\dfrac{d - x}{d} & \text{if } x \leq d, \end{cases} \tag{3}$$

where x is the distance to the wall and d the maximum distance at which the wall is "felt," while c stands for the strength of the repulsion.

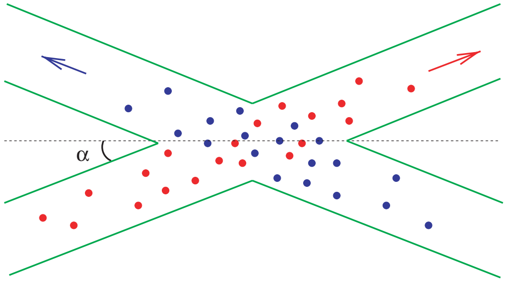

Fig. 1. The arrows represent the goals or driving forces of the agents.

The sum $\vec{f}_e = \vec{f}_w + \vec{f}_g$ has the role of an "external force" since it is the part of $\vec{f}$ which does not depend on the presence of other agents.

The "interaction" term of the decisional force is determined by the "observed" agents, i.e. those that fall in the field of view of the observer, which is a visual cone of given radius and angle centered on the agent's velocity (Fig. 2).

For each of these observed agents, the observer is supposed to know exactly the position, velocity and direction to the goal. The third assumption could seem unrealistic, but this information can be approximately deduced, in the case of real pedestrians, by observing the gaze or the "body language" of the other people (obviously an exact knowledge, not only of the goal but also of the position and velocity, is unrealistic, and is just one of the approximations of our model).

The logic at the basis of the decisional mechanism is to understand if there is the danger of a collision and to apply a force to avoid it. In order to do that the observer (agent i) examines all the relative positions and velocities of the observed agents ($\vec{r}_{ij}$ and $\vec{v}_{ij}$) and calculates the time at which the approaching agents (defined as those for which the angle between $\vec{r}_{ji}$ and $\vec{v}_{ij}$ is $\theta_{ij} < \frac{\pi}{4}$) will reach the minimum distance.

The minimum of these times,

$$t_{\text{pi}} = \min_{j \,\in\, \text{field of view},\ \theta_{ij} < \frac{\pi}{4}} \frac{r_{ij}\cos(\theta_{ij})}{v_{ij}}, \tag{4}$$

is defined as the "time of probable impact" (Fig. 3), at which the future positions of all the observed agents (both approaching and not) are calculated (Fig. 4).

The interaction force $\vec{f}_{\text{int}}$ is calculated as a sum of central repulsive forces depending on these future relative distances at the time of probable impact $\vec{d}_{ij}(t_{\text{pi}})$.

Each term will be given as (Fig. 5)

$$\vec{f}_{\text{int}}(\vec{d}) = \begin{cases} w(t_{\text{pi}}, v_{\text{pi}})\gamma\vec{e}_d & \text{if } d \le D_0, \\ w(t_{\text{pi}}, v_{\text{pi}})\gamma\left(\dfrac{d}{D_0}\right)^{-p}\vec{e}_d & \text{if } d > D_0, \end{cases} \tag{5}$$

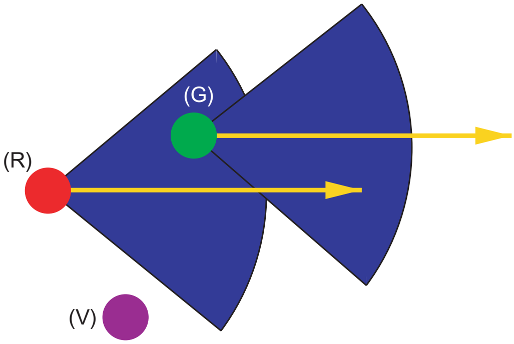

Fig. 2. Red (R) sees green (G) but does not see violet (V), while green does not see nothing. The arrows stand for the velocities.

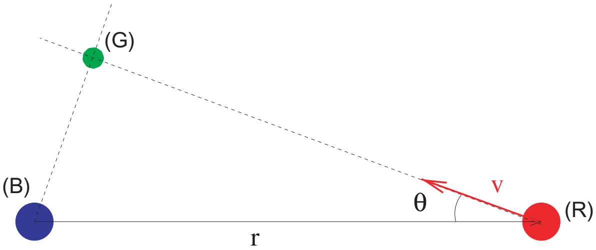

Fig. 3. The green spot (G) corresponds to the (future) position of minimal distance of the observed red agent (R) with respect to the blue observer (B).

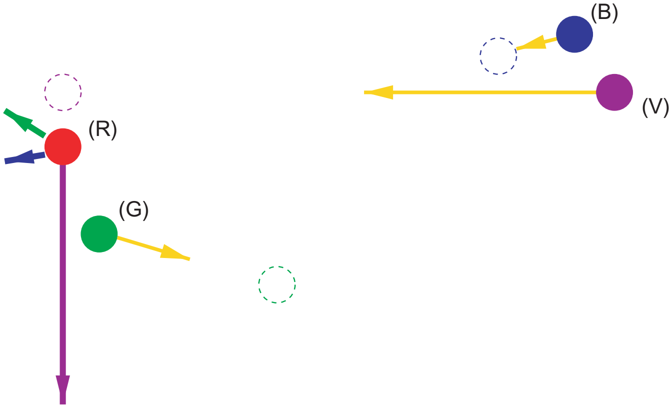

Fig. 4. Red (R) is the observer. t_{pi} is determined by violet (V), and at this time all the (future) positions of the observed agents, violet, green (G) and blue (B), are calculated. Red feels repulsion central forces determined by these future positions (dotted empty balls).

where $p > 0$ and $\gamma > 0$, v_{pi} is the velocity of the agent that is going to cause the "impact" at t_{pi} and w is a term that determines the "danger" of the situation as

$$w(t_{\mathrm{pi}}, v_{\mathrm{pi}}) = \min\left(\frac{v_{\mathrm{pi}}}{\gamma\, t_{\mathrm{pi}}}, 1\right), \tag{6}$$

where γ is the maximum force that the agent can apply. w is defined in such a way as to attain a complete stop in the case of a frontal impact (actually these formulae have to be slightly modified in order to take into account the presence of the driving force to the goal).

D_0 is to be interpreted as the perception of the size of the agent's own body (its diameter, or the sum of its radius and the other agent's radius, assumed to be equal), or as a "comfortable distance" to another agent.

The last point to be clarified is how the new position of the observed agents is calculated, which depends on the ToM level. If the agent has no ToM, the calculation just described will not be performed at all ($\vec{f}_{\mathrm{int}} = 0$), and the agent changes its motion just according to its goal (the "external force," and thus a level 0 agent is a

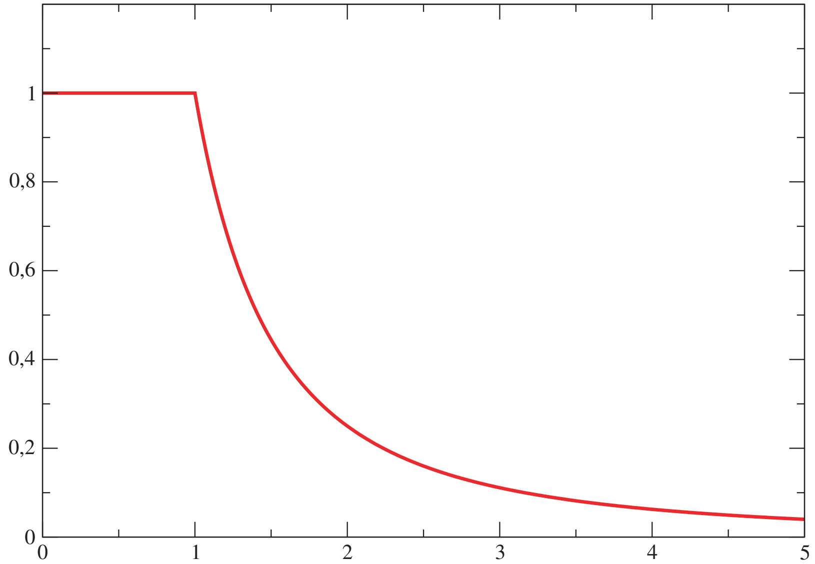

Fig. 5. Equation (5) for $D_0 = 1$, $p = 2$, $\gamma = 1$, $w = 1$.

noninteracting particle, or, more properly, it interacts only through elastic physical collisions, not through a decisional mechanism).

In the case of a level 1 agent, the other agents are assumed to be level 0, and their future position at time t_{pi} is calculated on the basis of their velocity corrected by the impulsive external force $\vec{f}_{\text{int}}$. As we said before, "following the gaze" the observer is able to know the goal of the observed agent, i.e. the direction of the driving force. But this does not mean that it knows the intensity of this force. Actually, in this first version of the model, each agent will assume that all the agents are identical to itself, and will consider that they are driven to their goal with the same attraction that it feels for its own, even if this is not necessarily the case.

Since we are talking about relative positions (following the discussion described in Fig. 4), to calculate them the observer has to consider its own future position too. This is calculated applying to itself the same ToM it applies to the others, i.e. assuming that it will act as a level 0 agent (even if it "knows" to be level 1, since obviously all the calculations about future positions have to undergo some approximation).

Notice that even in this first case a superposition principle does not apply. Due to the presence of t_{pi} the force that the observer feels when it sees simultaneously two agents is not the sum of the two forces felt when a single agent is observed.

In the case of an agent with a level 2 ToM (or a level $n > 1$ ToM), the observer performs all the level 1 (level $n-1$) calculations for all the observed agents (including itself). Obviously it performs these calculations on the basis of its own observations,

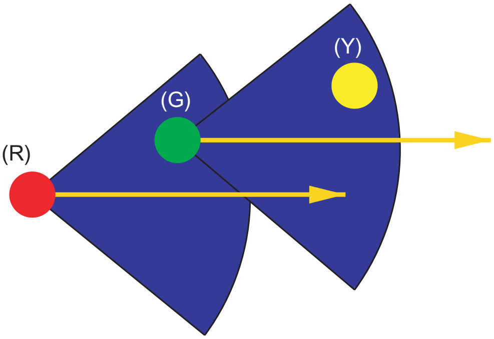

Fig. 6. Red (R) thinks that green (G) does not see nothing, while it sees yellow (Y).

which do not necessarily imply a perfect knowledge of the other agent's observations (see Fig. 6).

Notice also that, in the case of a level $n > 1$ agent, the superposition principle does not apply not only for the presence of the time of probable impact t_{pi}, but also because in this case the "future predicted positions" of the observed agents are modified by the presence of other agents (since these agents are considered by the observer to be at least level 1 and thus to interact with the others).

3. Main Features of the Model

Before proceeding to the complete study of the evolutionary process, we have verified that the model is able to reproduce, at least for quite a large range of parameters, some of the features of the observed organized behavior of actual pedestrians.

According to Ref. 9, when two fluxes of pedestrians with different goals cross (as in Fig. 1), the formation of stripes in the crossing region is observed. In the case of a single corridor with opposing goals (as when $\alpha = 0$ in Fig. 1), a flocking behavior with the formation of lanes is observed.

We show in Fig. 7 the typical behavior of the model in these conditions, which reproduces the desired effect. In these experiments, each time the agents reached the end of the corridor, they were recreated at the beginning of it, while losing any information about their transversal position and their velocity. This implies that the organization has to emerge at a local level, since no long-time or memory effect is present.

The results of Fig. 7 were obtained using level 1 agents. We have noticed that, following the results by Takano [6,7], an organized behavior is easier to obtain using odd ToM levels. We can explain this effect as a contrast between "careful" (odd) and "bold" (even) levels.

A level 0 agent is "bold" in the sense that it does not care about the others' behavior. A situation in which two level 0 agents have contrasting goals leads to a collision. Level 1 agents consider everyone (including themselves) to be level 0, and thus they predict the collision, and in order to avoid it their behavior is very

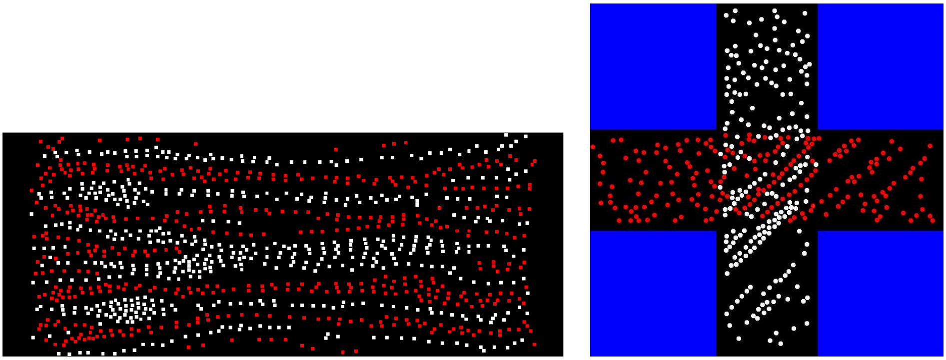

Fig. 7. Left: Formation of lanes in the case $\alpha = 0$. Right: Formation of stripes in the case $\alpha = \frac{\pi}{4}$.

"careful". But level 2 agents consider everyone to be very "careful," and thus "try to take advantage" of this situation behaving in a "bold" way.

The difference between odd and even levels is very strong and is probably due to the simplicity of the situation. Actual recursive thinking emerges probably due to complex social interactions, while to describe crowd dynamics a behavior based on "common sense" is surely more adequate. Nevertheless we think that this model is at the same time simple and realistic enough to perform an evolutionary simulation of the ToM level.

4. Parameters

The evolvable parameters ("genes") are: angle of view α_v, radius of view r_v, ToM level l, attraction to the goal f_g, "perceived diameter of body" D_0, interaction strength γ and exponent p [see Eq. (5)]. The results in Fig. 7 were obtained using agents with a physical radius $r = 0.005$, a maximum allowed velocity $v_{\max} = 0.1$ and an integration (decision) time $\Delta_t = 0.001$. $v_{\max}$ and r have been considered just as adimensional parameters, but they can be made dimensional and freely scaled to any desired values changing the time and length scales. All the other parameters in the model should also be scaled if dimensional (f_g and γ scale as forces, D_0 and r_v as lengths; masses have been considered fixed to 1).

The values for the visual cone were fixed to $r_v = 0.05$, $\alpha_v = \frac{\pi}{2}$, while the other parameters have been obtained by a preliminary application of genetic algorithm as $f_g = 0.97$, $\gamma = 0.24$, $D_0 = 0.01$ (interestingly, twice the physical radius), $p = 10$.

5. Conclusion

We have presented a collision-avoiding crowd dynamics model in order to perform an evolutionary simulation of some perceptional and behavioral parameters, as the level of a "theory of mind." We have shown that the model reproduces some of the basic features of the organized motion of pedestrians, and thus is well suited for an evolutionary simulation in a "realistic" physical environment.

References

[1] Kinderman, P., Dunbar and R., Bentall, R. P., Theory-of-mind deficits and causal attributions, *Brit. J. Psychol.* **89** (1998) 191–204.

[2] Miklósi, A., Polgádi, R., Topál, J. and Csányi, V., Use of experimenter-given cues in dogs, *Anim. Cogn.* **1** (1998) 113–121.

[3] Tschudin, A., Call, J., Dunbar, R., Harris, G. and Van der Elst, C., Comprehension of signs by dolphins (Tursiops truncates), *J. Comp. Psychol.* **115** (2001) 100–105.

[4] Kaminski, J., Riedl, J., Call, J. and Tomasello, M., Domestic goats (*Capra hircus*) follow gaze direction and use social cues in an object choice task, *Anim. Behav.* **69** (2001) 11–18.

[5] Bugnyar, T., Stöwe, M. and Heinrich, B., Ravens, Corvus corax, follow gaze direction of humans around obstacles, *Proc. Roy. Soc.* **271** (2004) 1331–1336.

[6] Takano, M., Kato, M. and Arita, T., A constructive approach to the evolution of the recursion level in the theory of mind, *Cogn. Stud.* **12**(3) (2005) 221–233 (in Japanese).

[7] Takano, M. and Arita, T., Asymmetry between even and odd levels of recursion in a theory of mind, *Proc. ALIFE X (10th Int. Conf. Simulation and Synthesis of Living Systems*, 2006), pp. 405–411.

[8] Helbing, D., Buzna, L., Johansson, A. and Werner, T., Self-organized pedestrian crowd dynamics: Experiments, simulations, and design solutions, *Trans. Sci.* **39** (2005) 1–24.

[9] Ando, K., Oto, H. and Aoki, T., Forecasting the flow of people, *Railway Res. Rev.* **45** (1988) 8–13 (in Japanese).